Formation of compositional structures by sedimentation in vigorous convection

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Abstract

Sedimentation is a mechanism in a convecting fluid that leads to differentiation and to the formation of compositional structures. The formation of layers is an important mechanism for both local features like magma chambers and for global features like the Earth’s mantle. Besides layering, sedimentation can lead to the formation of compositional heterogeneities at the bottom of a convective system. This may be potentially important for the dynamics of the lower mantle. We present a novel algorithm that enables us to numerically study sedimentation of finite-sized particles in non-dilute convective suspensions. Our approach considers a consistent settling velocity and the density contribution due to particle mass. We implement the settling algorithm into a 2D convection model and vary the buoyancy ratio \( B \) for five different Rayleigh numbers, covering a range of four orders of magnitude. We find \( B \) to be a critical parameter and its critical value to depend on the Rayleigh number. For subcritical values we observe that the presence of a crystal phase reduces convective vigor and most crystals stay suspended. Episodic layering can be seen for the critical value of \( B \). A supercritical buoyancy ratio forces layering on the system, which we find to be applicable to convective magma chambers. Layer formation in all observed layering cases occurs on a time scale that is short compared to the time that magma chambers take to solidify. We find that dynamical layer formation that is connected to crystal settling and the crystals’ density contribution is a likely mechanism for creating layered structures within the convective lifetime of a magma chamber. Additionally, we observe that when the forces are appropriately balanced most of the crystals remain suspended while compositional heterogeneities accumulate at the lower boundary.

Keywords: Structure formation; Compositional heterogeneity; Convection; Sedimentation; Numerical modeling

1. Introduction

Particle settling in a convecting fluid has received much attention for its significance as an efficient differentiation mechanism in magma chambers and, on a global scale, in an early terrestrial magma ocean (e.g. Marsh and Maxey, 1985; Martin and Nokes, 1988, 1989; Huppert et al., 1991; Sparks et al., 1993; Solomatov and Stevenson, 1993). Differentiation processes can create a myriad of spatial structures. While layering is the most prominent structure appearing in magma chambers less regular compositional structures may develop from the interplay of convection and sedimentation.

Compositional heterogeneities in the lowermost part of the mantle and especially at the core–mantle boundary have been proposed in order to explain observations from seismology (van der Hilst et al., 1991), geochemistry (Christensen and Hofmann, 1994) as well as the heat budget of mantle convection (Kellogg et al., 1999). Recent seismological investigations indicate that compositional effects significantly influence the seismic wave speed and possibly the buoyancy distribution of the man-
tle flow (Tramper et al., 2004). The majority of work on the dynamics of compositional structures in the convecting lower mantle has been done using numerical models (Hansen and Yuen, 1988, 2000; Tackley, 1998; Davies, 2002; Nakagawa and Tackley, 2004). Many of the models make assumptions about the origin of such compositional structures and range from chemical reactions between the core and the mantle to a relict layer left over from core formation (Hansen and Yuen, 2000). In this paper, we investigate particle settling with respect to emergent structures, resulting from the competing effects of convection and settling.

Much of the previous work focused on very dilute suspensions and thus neglected the influence of the particles on the convective flow. However, Koyaguchi et al. (1990) reported that already a very small particle concentration (∼0.3 wt.%) suppresses turbulent convection and significantly modifies the flow. The authors suggest that not only the settling velocity but also the buoyancy ratio, which is the ratio of the density variation due to particles to the thermal density variation, has to be taken into account.

Numerical studies have assumed that regions of closed particle trajectories in steady convection coincide with areas of suspension (Weinstein et al., 1988; Rudman, 1992). The applicability to unsteady flows however seems problematic (Solomatov and Stevenson, 1993). Several conceptually different methods have been applied to model a crystal phase in thermo-chemical convection, of which the tracer approach is reported to be the most flexible and robust method (van Keken et al., 1997). The main advantage of a tracer method compared to a field approach is the capacity to model zero diffusivity. This is important at steep gradients where even small diffusivities can cause artificially high fluxes. A number of tracer based models have been developed to date (e.g. Christensen and Hofmann, 1994; Tackley and King, 2003). However, to our knowledge there is no numerical study involving tracers that considers the settling velocity in a non-dilute suspension.

Here, we present a new algorithm that enables us to study sedimentation of finite-sized crystals in non-dilute convective suspensions, which is of great importance for investigating magma chambers. It also allows the study of structure generation on a global scale that is connected to Earth’s differentiation or the thermo-chemical evolution of a terrestrial magma ocean. Solomatov and Stevenson (1993) point out that the crystal’s presence significantly influences the style of differentiation of a terrestrial magma ocean.

2. Model and methods

Convecting two-phase flows at infinite Prandtl number can represent magma chamber convection (Rudman, 1992). The governing equations for thermo-chemically driven convection in an infinite Prandtl-number Boussinesq fluid in a two-dimensional Cartesian geometry are (Tramper and Hansen, 1996):

\[ \nabla \cdot \dot{\varepsilon} - \nabla P - Ra(T - BC) \dot{\varepsilon} = 0 \tag{1} \]

\[ \frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla) T - \nabla^2 T = 0 \tag{2} \]

\[ \nabla \cdot \vec{u} = 0 \tag{3} \]

Here, \( \dot{\varepsilon} \), \( \vec{u} \), \( t \), \( P \) and \( T \) are the deviatoric strain rate tensor, velocity, thermal diffusion time, pressure and temperature, respectively. The Rayleigh number

\[ Ra = \frac{\alpha g \Delta T d^3}{\kappa v} \tag{4} \]

expresses the ratio of buoyancy forces to viscous and diffusive forces, where \( \alpha \), \( g \), \( v \) and \( \kappa \) respectively denote the thermal expansion coefficient, the acceleration due to gravity, the kinematic viscosity and the thermal diffusivity. \( d \) is the length scale (height) of the Cartesian geometry over which the temperature difference \( \Delta T \) is maintained. The buoyancy ratio \( B \) is the ratio of the density variation due to \( C \) to the thermal density variation:

\[ B = \frac{\Delta \rho_C}{\rho_C \Delta T} \tag{5} \]

The numerical scheme employed here is two-fold. It consists of a finite-volume based convection model that solves Eqs. (1)–(3) (Tramper and Hansen, 1996) and a tracer module, which advances and settles the tracers to simulate crystal movement. The tracer distribution is used to compute the concentration field \( C \), which in turn affects the buoyancy term in Eq. (1).

The temperature is held fixed at the upper \((T = 0)\) and lower \((T = 1)\) boundary. There are no flux boundary conditions at the vertical boundaries. All boundaries are stress-free and impermeable for tracer mass.

Particles with negligible inertia move with a velocity that is equal to the sum of the local fluid’s velocity plus a settling velocity. Therefore, we split up the computation of a tracer’s trajectory into two parts. First a tracer is advected using a fourth-order Runge–Kutta scheme with a bilinear spatial interpolation of the local fluids velocity, the accuracy of which has been verified by Schmalzl et al. (1995). The particle’s location is then updated using the Stokes’ settling velocity \( v_s \) in analogy to a free-falling
spherical particle (e.g. Davis and Acrivos, 1985):

\[ v_s = \frac{2a'^2 \delta \rho g}{9 \rho v F} f(C) \]  

(6)

where \( a' \) is the particles’ radius and \( \delta \rho \) is the density difference of one particle to the ambient fluid. The hindered settling function \( f(C) \) considers the hindered settling of individual particles by the presence of neighboring particles and by the necessary upward return flow of interstitial fluid (Huppert et al., 1991). To compute \( f(C) \) we use the formula proposed by Richardson and Zaki (1954):

\[ f(C) = (1 - C)^n \]  

(7)

with an empirical value of \( n = 5.1 \) suggested by Garside and Al-Dibouni (1977). Only particles in clear ambient \( C = 0 \) will thus settle with the full settling velocity \( v_s \).

In order to avoid unphysically high particle concentrations we utilize the newly developed settling algorithm that takes the particles finite size into account to restrict the number of tracers per control volume. A very fine equidistant grid divides the computational domain into squares, each of which represents the spatial extent of one particle. This grid is typically 25–400 times finer than the spatial size of the control volumes used to solve the equations that govern the convection. Thus a maximal particle density is defined which corresponds to \( C = 1 \). This grid holds only the binary information whether or not a particle is contained inside a given cell. For each time step, after solving Eqs. (1)–(3), we use the heapsort algorithm to sort the tracers by vertical location. After that we update the binary fine-grid information. Then, starting with the tracer closest to the bottom, we compute its settling distance using the product of its settling velocity \( v_s \) and the current time step \( dt \). If this distance exceeds the fine-grid cell size the particle settles downwards successively, visiting one cell after another. If a target cell is not filled, the particle’s spatial vertical coordinate \( z \) is updated and the cell is marked as being full. The two cells on either side are checked if during the last iteration the target cell is filled. If none of them are vacant the particle remains at its current position. If one of the neighboring cells is empty the particle is moved inside by updating its coordinates accordingly. Doing this avoids creating unphysically tall stacks of particles.

Many authors use the ratio of the particle settling velocity to the convective velocity \( \langle S \rangle \) as a free parameter. We chose a different approach. Rather than specifying the velocity ratio \( \delta \) we specify the particle radius \( a' \) and compute the settling velocity \( v_s \) (Eq. (6)).

The average concentration \( \langle C \rangle \) is another free parameter since it relates to the total number of particles \( N_0 \) used in the simulation. Therefore, \( \langle C \rangle \) is a measure of how much particle mass is contained in the convecting fluid. For all numerical experiments presented in this work we keep the dimensionless particle radius \( a = a'/d \) and \( \langle C \rangle \) fixed at \( a = 1/640 \) and \( \langle C \rangle = 0.4 \).

The set of dimensionless free parameters used in this model consists of \( Ra, B, a, \langle C \rangle \) as well as the Prandtl-number.

We will now demonstrate how we compute \( C \) and \( \langle C \rangle \). The computational domain is divided into \( nx \times nz \) volumes of size \( dx \times dz \) such that \( d = dx \times dz = nz \). \( N_{ij} \) is the number of particles in such a volume, where \( i \in [1, nx] \) and \( j \in [1, nz] \). The total number of particles is then

\[ N_0 = \sum_{i=1}^{nx} \sum_{j=1}^{nz} N_{ij} \]  

(8)

By using a new settling algorithm described above the number of particles per volume is restricted to

\[ N_{max} = \frac{dx dz}{(2a')^2} = \left(4a'^2 nx nz \right)^{-1} \]  

(9)

The particle concentration \( C \) is computed as \( C_{ij} = N_{ij}/N_{max} \) leading to \( C \in [0 : 1] \). The average concentration is

\[ \langle C \rangle = \int_0^d \int_0^d C(x, z) dx dz \rightarrow \frac{1}{d^2} \sum_{i=1}^{nx} \sum_{j=1}^{nz} C_{ij} \frac{d}{nx nzw} \]

\[ = \frac{1}{d^2} \sum_{i=1}^{nx} \sum_{j=1}^{nz} N_{ij} 4a'^2 nx nz \frac{d}{nx nzw} = 4a'^2 N_0 \]  

(10)

The model uses a constant viscosity and only considers particle–particle interactions in the hindered settling function and in the settling algorithm. This approximation becomes less accurate as \( C \) approaches 1.

3. Performance

In order to estimate the numerical demand for high resolution applications we investigate how the computation time for a time step scales with the spatial resolution of the problem geometry \( n = nx \times nz \). The Cartesian domain is divided into \( nx \) and \( nz \) control volumes in horizontal and vertical direction, respectively. We use \( 10n \) tracers so that the tracer-based field \( C \) is resolved equally well with varying \( n \). To ensure that the average tracer concentration \( \langle C \rangle \) is constant for each case, we vary the particle radius \( a \) accordingly.

We perform a series of calculations where we vary \( n \) and measure the CPU time needed by a Pentium 4 to solve the Eulerian fluid flow problem and the Lagrangian tracer problem, respectively. The tracer algorithm, as
number. The investigated Rayleigh numbers are $Ra = 10^4, 5 \times 10^4, 10^5, 10^6$ and $10^7$. The flow structures are well captured by the $64 \times 64 (128 \times 128$ for $Ra = 10^7)$ spatial resolution which allows for at least four grid points inside the boundary layers. We tracked 40,960 tracers as to have an average of 10 tracers per control volume. Note that our settling algorithm limits the number of tracers per cell which avoids tracer clumping. This has resulted in severe restrictions for the viability of many tracer methods (Tackley and King, 2003). We chose the particle radius to be $a = 1/640$ such that the maximal number of tracers per control volume is 25, which corresponds to $C = 1$. Initially there is no stratification in the system, i.e. $T = 0, C = \langle C \rangle = 0.4$.

The amount of sedimentation can be preeminently captured by the relative entrainment $e$, defined as the ratio of the number of particles contained in the upper half of the cell to the total number of particles:

$$e = \frac{\int_{d/2}^{d} \int_{0}^{d} C \, dx \, dz}{\int_{0}^{d} \int_{0}^{d} C \, dx \, dz}$$

The initial state at which the particles are distributed randomly throughout the cell yields $e = 1/2$. The time it takes a crystal to travel across the height of the cell with the full settling velocity in the absence of a thermal gradient defines the settling timescale $t_s$:

$$t_s = \frac{d}{v_s}$$

We will present our results in terms of this timescale. For each set of Rayleigh numbers we find a critical value of $B$ that separates crystal suspension from layer formation.

4.1. Layer formation

Fig. 2 shows a typical case of layer formation ($Ra = 10^6, B = 8$). A layer free of particles forms at the top due to particle settling while a sediment layer forms at the bottom. The interfacial layer decreases over time as all crystals settle out. The time history of the relative entrainment for this case is shown in Fig. 3. It can be seen that only a small fraction of particles stay suspended in the upper half. Layer formation is observed when the presence of particles dominates the fluid flow. The initially unstratified system evolves into a stratified one, where both layers convect with very little vertical flux across their interface. This can be seen from the temperature field and its profile in Fig. 2d. For $Ra < 10^6$ the bottom layer does not convect. All layers are formed within 0.9–2.6 settling times, i.e. layer formation occurs on the time scale of $t_s$. 

Fig. 1. Comparison of computing time required to solve the Eulerian problem (fluid flow) and the Lagrangian problem (particles), respectively, for different spatial resolutions. The number of tracers is proportional to $n$.

Fig. 1 shows the CPU time used for solving the Eulerian problem (fluid flow) and the Lagrangian problem (particles), respectively. The data is averaged over 1000 time steps. Both problems scale with the spatial resolution $n$, which allows for a prediction for larger $n$. The time used for the tracer treatment is on average 39% of the total time needed to compute a time step. Thus it is on the same order of magnitude as the time required to compute the fluid flow. We conclude that the settling algorithm presented here is a feasible method to investigate sedimentation of finite-sized particles in a non-dilute convective suspension. This is true even for applications that require a large number of control volumes, like high-resolution and three-dimensional computations. Furthermore, the scaling properties provided in Fig. 1 allow for an accurate estimate of the computation time.

4. Results

We performed five sets of numerical experiments where we varied the buoyancy ratio for a given Rayleigh
Fig. 2. Snapshots of temperature $T$ and concentration $C$ as well as their profiles for a layering case ($Ra = 10^6, B = 8$) at different times (a) $t = 0$, (b) $t = 0.4t_s$, (c) $t = 1.4t_s$, (d) $t = 4.3t_s$. The snapshot color scale is from blue ($T = 0, C = 0$) via green to red ($T = 1, C = 1$). The profiles indicate three distinct layers with varying thicknesses: a particle-free top layer, an interface layer and a sediment layer at the bottom.

4.2. Suspension

An example of a suspension case ($Ra = 10^6, B = 2$) can be seen in (Fig. 4), which shows the temperature, the concentration of particles and their horizontally averaged profiles at different times. Fig. 4d shows this case after 1.6 settling times. This corresponds to the order of 400 overturn times, where the overturn time is defined as the revolution time of a particle following a streamline in steady thermal convection.

For subcritical buoyancy ratios we find most of the particles to be advected with the large scale flow and a small fraction to accumulate in piles at the floor of the cell. These structures consist of compositionally different material than the bulk. They are swept across the bottom boundary by the large scale flow while some reentrainment occurs at the tip of the piles. After a short transition period ($t < 0.1t_s$) the amount of reentrainment is matched by the amount of sedimenting material. The result is a constant time-averaged value for the relative entrainment, visible in its time history (Fig. 3), which is slightly below $1/2$ due to the material that has accumulated at the lower boundary.

A reduction of convective vigor can be quantified by the reduction of the Nusselt number, which measures the ratio of the total heat flow to the conductive heat flow. For $Ra = 10^6$ we find the time-averaged Nusselt number for convection with suspended heavy crystals ($B = 2$) to
be about 20% lower than the value for the steady-state purely thermal case, which is the case without particle mass. The decrease in the heat transport efficiency is likely due to the decrease of the rms velocity and possibly linked to particles covering the floor.

4.3. Recurrent layering

For the critical buoyancy number in the set of experiments with $Ra = 10^6$ we observe recurring layers. After layer formation due to sedimentation, material gets entrained by the large scale flow in the upper layer. The rate of reentrainment exceeds the settling rate and the layer begins to erode. When a substantial amount of particle mass is reentrained the cycle is repeated. Fig. 5 shows the time history of the relative entrainment $e$ for this case. A characteristic time scale of about 2.5 settling times can be identified.

4.4. Dependence of the critical buoyancy number on the Rayleigh number

Each set of experiments yields a critical buoyancy number, above which layering is observed. We find the critical value of $B$ to depend on the Rayleigh number and expect it to also depend on the particle size $a$ and the initial particle concentration ($C$). The latter number influences the predominance of the particle phase in the buoyancy term in Eq. (1). The particle size $a$ is a crucial parameter because it enters the settling velocity to the second power.

In Fig. 6, we show the investigated phase space spanned by the buoyancy ratio and the Rayleigh number. A line through the critical buoyancy ratios suggests where the suspension and the layering regimes might be located for our choices of $a$ and ($C$).

Although the obtained values for the critical buoyancy ratios are not universal it can be seen from Fig. 6 that increasing $Ra$ for a given $B$ can lead from a suspension regime into a settling regime.

As Eq. (1) shows the flow is driven by buoyancy variations only. The driving term in our formulation is $Ra(T - BC)$ using the Rayleigh number $Ra$ and the buoyancy number $B$, which are defined in Eqs. (4) and (5), respectively. An equivalent formulation for the buoyancy term uses the thermal Rayleigh number $Ra_T = Ra$ and a concentration Rayleigh number $Ra_C = Ra_T B$, leading to $Ra_T T - Ra_C C$ for the driving term. It is clear from this formulation that the overall buoyancy contribution due $C$ is connected to the product of $Ra$ and $B$.

5. Validation

We have presented a plot of the investigated phase space that suggests two regimes: one of limited particle influence, where a large fraction of particles stays suspended, and one of strong influence, where the particle’s density contribution forces layering onto the convective flow. For all investigated Rayleigh numbers we find a critical buoyancy ratio, above which layering can be achieved from an initially unstratified fluid.

Laboratory experiments of particle-laden fluids that are cooled from above show different regimes of convective motion, depending on the particles’ density contribution (Koyaguchi et al., 1990). Their theoretical analysis suggests that critical conditions are predominantly determined by the buoyancy ratio.

Furthermore, the competition between thermally destabilizing and compositionally stabilizing density influences have been seen to create repetitive cycles of sedimentation and reentrainment (Sparks et al., 1993), similar to the recurrent layering case presented here.

Although in their laboratory setup high Rayleigh number convection was the result of only cooling from above, the observation of such regimes verifies the viability of our numerical approach. Finding all three regimes in our studies indicates that our model is able to distinguish physically different scenarios.

We conclude that our novel settling algorithm is able to describe particle settling and suspension. Possible applications can be found on a variety of scales with magma chambers and a terrestrial magma ocean being only two examples. It should be noted that the particle settling method can be used in combination with any fluid flow model as is it not limited to convective flows.
Fig. 4. Snapshots of temperature $T$ and concentration $C$ as well as their profiles for a suspension case ($Ra = 10^6$, $B = 2$) at different times (a) $t = 0$, (b) $t = 0.001t_\alpha$, (c) $t = 0.02t_\alpha$, (d) example of statistical steady state, $t = 1.6t_\alpha$. The snapshot color scale is from blue ($T = 0$, $C = 0$) via green to red ($T = 1$, $C = 1$). A small bed of particles is formed at the bottom of the cell while most of the crystals are suspended.

6. Application to layer formation in magma chambers

Results from implementing our new method into a two-dimensional convection model indicate that the presence of particles (e.g. crystals in magmatic environments) can significantly alter convective motion when a critical buoyancy ratio is exceeded. For all investigated Rayleigh numbers we find a critical buoyancy ratio above which stable layers form dynamically.

Most of the crystal mass collects in the bottom layer, even for cases where the average settling velocity is three orders of magnitude smaller than the root mean square convective velocity. Layer formation occurs on the time scale connected to the full settling velocity, which is short compared to the time that magma chambers take to solidify (Martin and Nokes, 1989).

Crystal settling is thus a potential mechanism of dynamical layer formation during the convective lifetime of a magma chamber, provided that the crystals’ density contribution is supercritical. Here we neglect secular cooling of magma chambers, which involves phase transitions, crystal nucleation and crystal growth. Including these effects would lead to an increase in the particles’ density contribution. Thus, during the cooling time of a magma chamber, an evolution towards conditions that favor layer formation is very likely even for cases that are initially subcritical.
The model’s stress-free boundary conditions do not resemble the rigid boundaries of magma chambers. However, for supercritical buoyancy numbers we observe that the stabilizing effect of suspended crystals dominates over the destabilizing temperature gradient within the bulk of the flow. This sufficiently decreases convective motion and the related uplift of crystals, leading to crystal sedimentation throughout the flow and to subsequent layer formation in a stress-free boundary configuration. It is reasonable to assume that rigid boundary conditions would increase this effect.

We conclude that in connection with other sedimentation mechanisms at the rigid bottom boundaries of magma chambers, crystal settling and the crystals’ density contribution can be responsible for layered structures observed in magma chambers.

Further work is needed to more fully understand the complicated dynamics inside magma chambers. In particular higher Rayleigh number flows with temperature and crystal-content dependent viscosity as well as the inclusion of secular cooling, crystal nucleation and growth should receive closer attention.

7. Application to the core–mantle boundary

Seismologists have found the lowermost part of the mantle to have distinct properties from the overlying mantle. This region, $D''$, is characterized by a shear wave discontinuity 150–450 km above the core–mantle boundary and a reduced velocity gradient directly below the discontinuity. The lateral lengthscale ranges from 250 km to several 1000 km. It has been suggested that $D''$ is thermally and chemically distinct from the mantle.

A different point of view originates from a numerical study by Davies (2002), who finds gravitational settling of dense tracers at the base of the mantle-representing convection cell. The author infers that the change in tomographic character in the deep mantle can possibly be explained without the need to postulate a separate layer.

Most other work on $D''$ has assumed pre-existing layers and has investigated their stability (e.g. Hansen and Yuen, 1988; Tackley, 1998; Samuel and Farnetani, 2003). The formation of the sharp seismic discontinuity at the core–mantle boundary remains an important yet unresolved problem (Tolstikhin and Hofmann, 2005). Three possible origins for the $D''$ layer have been proposed so far: (1) formation during the initial differentiation of the Earth (e.g. Olson and Kincaid, 1991), (2) formation by subducted basaltic crust (e.g. Tolstikhin and Hofmann, 2005), and (3) formation due to chemical reactions with the core (e.g. Kellogg and King, 1993).

Here we present a fourth mechanism that can lead to the formation of a compositionally distinct layer at the core–mantle boundary. The suspension scenario presented here shows a mechanism for segregating compositionally distinct material. For subcritical buoyancy ratios most crystals remain suspended while small dynamic piles form at the bottom of an initially unstratified system. The creation of compositionally distinct reservoirs at the base of the mantle is a direct result of the dynamics of thermo-chemical convection when the forces are appropriately balanced.
It is worth noting that the compositional structures accumulate in patches at the lower boundary of the convective cell and not as a continuous layer. This has consequences for the effectiveness of reentrainment as entrainment by viscous coupling between crystals and the ambient fluid can be more effective in localized structures (Solomatov et al., 1993).

The scenario presented here is clearly oversimplified and should be revisited in a more detailed study. Considering laterally varying viscosity and effects of internal heating could likely enhance the understanding of topography and evolution of compositional heterogeneities in the deep mantle.

However, this case of sedimentation in convection demonstrates a mechanism that could have contributed to the formation of a compositional heterogeneity at the core–mantle boundary.

8. Conclusions and outlook

We have introduced a new algorithm for the numerical study of finite-sized-particle settling in a non-dilute system. This method prevents the unphysical clustering of particles which has been a reported problem in tracer approaches that simulate heavy suspended particles. As a tracer based method it also allows modeling of zero diffusion in the crystal phase which has been the major disadvantage of field approaches.

We have presented the implementation of the new method into a two-dimensional convection model and have shown how this method can be applied to both small scale features like magma chambers and large scale systems like the Earth’s mantle. For less viscous objects, especially those strongly influenced by Coriolis forces like a terrestrial magma ocean, the two-dimensional representation imposes too much of a restriction. Only a three-dimensional model that includes rotation can accurately describe a flow structure expected in a rotating large scale system.

The introduction of the algorithm into a three-dimensional model is straightforward and feasible with regard to its scaling properties. We are pursuing its implementation into an existing three-dimensional model that allows us to consider effects of inertia and rotation in order to study key mechanisms that might give insights into the dynamics of a terrestrial magma ocean.

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